An Introduction To Riemannian Geometry With Applications To Mechanics And Relativity Universitext

This text employs vector methods to explore the classical theory of curves and surfaces. Topics include basic theory of tensor algebra, tensor calculus, calculus of differential forms, and elements of Riemannian geometry. 1959 edition.

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This classic text serves as a tool for self-study; it is also used as a basic text for undergraduate courses in differential geometry. The author's ability to extract the essential elements of the theory in a lucid and concise fashion allows the student easy access to the material and enables the instructor to add emphasis and cover special topics. The extraordinary wealth of examples within the exercises and the new material, ranging from isoperimetric problems to comments on Einstein's original paper on relativity theory, enhance this new edition.

This book introduces advanced undergraduates to Riemannian geometry and mathematical general relativity. The overall strategy of the book is to explain the concept of curvature via the Jacobi equation which, through discussion of tidal forces, further helps motivate the Einstein field equations. After addressing concepts in geometry such as metrics, covariant differentiation, tensor calculus and curvature, the book explains the mathematical framework for both special and general relativity. Relativistic concepts discussed include (initial value formulation of) the Einstein equations, stress-energy tensor, Schwarzschild space-time, ADM mass and geodesic incompleteness. The concluding chapters of the book introduce the reader to geometric analysis: original results of the author and her undergraduate student collaborators illustrate how methods of analysis and differential equations are used in addressing questions from geometry and relativity. The book is mostly self-contained and the reader is only expected to have a solid foundation in multivariable and vector calculus and linear algebra. The material in this book was first developed for the 2013 summer program in geometric analysis at the Park City Math Institute, and was recently modified and expanded to reflect the author's experience of teaching mathematical general relativity to advanced undergraduates at Lewis & Clark College.

This book is an exposition of semi-Riemannian geometry (also called pseudo-Riemannian geometry)--the study of a smooth manifold furnished with a metric tensor of arbitrary signature. The principal special cases are Riemannian geometry, where the metric is positive definite, and Lorentz geometry. For many years these two geometries have developed almost independently: Riemannian geometry reformulated in coordinate-free fashion and directed toward global problems, Lorentz geometry in classical tensor notation devoted to general relativity. More recently, this divergence has been reversed as physicists, turning increasingly toward invariant methods, have produced results of compelling mathematical interest.

This textbook for graduate students is intended as an introduction to differential geometry with principal emphasis on Riemannian geometry. Chapter I explains basic definitions and gives the proofs of the important theorems of Whitney and Sard. Chapter II deals with vector fields and differential forms. Chapter III addresses integration of vector fields and p-plane fields. Chapter IV develops the notion of connection on a Riemannian manifold considered as a means to define parallel transport on the manifold. The author also discusses related notions of torsion and curvature, and gives a working knowledge of the covariant derivative. Chapter V specializes on Riemannian manifolds by deducing global properties from local properties of curvature, the final goal being to determine the manifold completely. Chapter VI explores some problems in PDEs suggested by the geometry of manifolds. The author is well known for his significant contributions to the field of geometry and PDEs--particularly for his work on the Yamabe problem--and for his expository accounts on the subject. The text contains many problems and solutions, permitting the reader to apply the theorems and to see concrete developments of the abstract theory.

This book provides an introduction to Riemannian geometry, the geometry of curved spaces, for use in a graduate course. Requiring only an understanding of differentiable manifolds, the author covers the introductory ideas of Riemannian geometry followed by a selection of more specialized topics. Also featured are Notes and Exercises for each chapter, to develop and enrich the reader's appreciation of the subject. This second edition, first published in 2006, has a clearer treatment of many topics than the first edition, with new proofs of some theorems and a new chapter on the Riemannian geometry of surfaces. The main themes here are the effect of the curvature on the usual notions of classical Euclidean geometry, and the new notions and ideas motivated by curvature itself. Completely new themes created by curvature include the classical Rauch comparison theorem and its consequences in geometry and topology, and the interaction of microscopic behavior of the geometry with the macroscopic structure of the space.

Provides a comprehensive and self-contained introduction to sub-Riemannian geometry and its applications. For graduate students and researchers.

The series is devoted to the publication of monographs and high-level textbooks in mathematics, mathematical methods and their applications. Apart from covering important areas of current interest, a major aim is to make topics of an interdisciplinary nature accessible to the non-specialist. The works in this series are addressed to advanced students and researchers in mathematics and theoretical physics. In addition, it can serve as a guide for lectures and seminars on a graduate level. The series de Gruyter Studies in Mathematics was founded ca. 30 years ago by the late Professor Heinz

Bauer and Professor Peter Gabriel with the aim to establish a series of monographs and textbooks of high standard, written by scholars with an international reputation presenting current fields of research in pure and applied mathematics. While the editorial board of the Studies has changed with the years, the aspirations of the Studies are unchanged. In times of rapid growth of mathematical knowledge carefully written monographs and textbooks written by experts are needed more than ever, not least to pave the way for the next generation of mathematicians. In this sense the editorial board and the publisher of the Studies are devoted to continue the Studies as a service to the mathematical community. Please submit any book proposals to Niels Jacob.

Many problems in the sciences and engineering can be rephrased as optimization problems on matrix search spaces endowed with a so-called manifold structure. This book shows how to exploit the special structure of such problems to develop efficient numerical algorithms. It places careful emphasis on both the numerical formulation of the algorithm and its differential geometric abstraction--illustrating how good algorithms draw equally from the insights of differential geometry, optimization, and numerical analysis. Two more theoretical chapters provide readers with the background in differential geometry necessary to algorithmic development. In the other chapters, several well-known optimization methods such as steepest descent and conjugate gradients are generalized to abstract manifolds. The book provides a generic development of each of these methods, building upon the material of the geometric chapters. It then guides readers through the calculations that turn these geometrically formulated methods into concrete numerical algorithms. The state-of-the-art algorithms given as examples are competitive with the best existing algorithms for a selection of eigenspace problems in numerical linear algebra. Optimization Algorithms on Matrix Manifolds offers techniques with broad applications in linear algebra, signal processing, data mining, computer vision, and statistical analysis. It can serve as a graduate-level textbook and will be of interest to applied mathematicians, engineers, and computer scientists.

This book provides a comprehensive account of a modern generalisation of differential geometry in which coordinates need not commute. This requires a reinvention of differential geometry that refers only to the coordinate algebra, now possibly noncommutative, rather than to actual points. Such a theory is needed for the geometry of Hopf algebras or quantum groups, which provide key examples, as well as in physics to model quantum gravity effects in the form of quantum spacetime. The mathematical formalism can be applied to any algebra and includes graph geometry and a Lie theory of finite groups. Even the algebra of 2 x 2 matrices turns out to admit a rich moduli of quantum Riemannian geometries. The approach taken is a `bottom up’ one in which the different layers of geometry are built up in succession, starting from differential forms and proceeding up to the notion of a quantum `Levi-Civita’ bimodule connection, geometric Laplacians and, in some cases, Dirac operators. The book also covers elements of Connes’ approach to the subject coming from cyclic cohomology and spectral triples. Other topics include various other cohomology theories, holomorphic structures and noncommutative D-modules. A unique feature of the book is its constructive approach and its wealth of examples drawn from a large body of literature in mathematical physics, now put on a firm algebraic footing. Including exercises with solutions, it can be used as a textbook for advanced courses as well as a reference for researchers.

This book deals with such important subjects as variational methods, the continuity method, parabolic equations on fiber bundles, ideas concerning points of concentration, blowing-up technique, geometric and topological methods. It explores important geometric problems that are of interest to many mathematicians and scientists but have only recently been partially solved.

Book endorsed by the Sunyer Prize Committee (A. Weinstein, J. Oesterle et. al.).

This text presents a graduate-level introduction to differential geometry for mathematics and physics students. The exposition follows the historical development of the concepts of connection and curvature with the goal of explaining the Chern–Weil theory of characteristic classes on a principal bundle. Along the way we encounter some of the high points in the history of differential geometry, for example, Gauss' Theorema Egregium and the Gauss–Bonnet theorem. Exercises throughout the book test the reader's understanding of the material and sometimes illustrate extensions of the theory. Initially, the prerequisites for the reader include a passing familiarity with manifolds. After the first chapter, it becomes necessary to understand and manipulate differential forms. A knowledge of de Rham cohomology is required for the last third of the text. Prerequisite material is contained in author's text An Introduction to Manifolds, and can be learned in one semester. For the benefit of the reader and to establish common notations, Appendix A recalls the basics of manifold theory. Additionally, in an attempt to make the exposition more self-contained, sections on algebraic constructions such as the tensor product and the exterior power are included. Differential geometry, as its name implies, is the study of geometry using differential calculus. It dates back to Newton and Leibniz in the seventeenth century, but it was not until the nineteenth century, with the work of Gauss on surfaces and Riemann on the curvature tensor, that differential geometry flourished and its modern foundation was laid. Over the past one hundred years, differential geometry has proven indispensable to an understanding of the physical world, in Einstein's general theory of relativity, in the theory of gravitation, in gauge theory, and now in string theory. Differential geometry is also useful in topology, several complex variables, algebraic geometry, complex manifolds, and dynamical systems, among other fields. The field has even found applications to group theory as in Gromov's work and to probability theory as in Diaconis's work. It is not too far-fetched to argue that differential geometry should be in every mathematician's arsenal.

This book introduces readers to the living topics of Riemannian Geometry and details the main results known to date. The results are stated without detailed proofs but the main ideas involved are described, affording the reader a sweeping panoramic view of almost the entirety of the field. From the reviews "The book has intrinsic value for a student as well as for an experienced geometer. Additionally, it is really a compendium in Riemannian Geometry." --MATHEMATICAL REVIEWS

An introduction to semi-Riemannian geometry as a foundation for general relativity Semi-Riemannian Geometry: The Mathematical Language of General Relativity is an accessible exposition of the mathematics underlying general relativity. The book begins with background on linear and multilinear algebra, general topology, and real analysis. This is followed by material on the classical theory of curves and surfaces, expanded to include both the Lorentz and Euclidean signatures. The remainder of the book is devoted to a discussion of smooth manifolds, smooth manifolds with boundary, smooth manifolds with a connection, semi-Riemannian manifolds, and differential operators, culminating in applications to Maxwell's equations and the Einstein tensor. Many worked examples and detailed diagrams are provided to aid understanding. This book will appeal especially to physics students wishing to learn more differential geometry than is usually provided in texts on general relativity.

This book aims to bridge the gap between probability and differential geometry. It gives two constructions of Brownian motion on a Riemannian manifold: an extrinsic one where the manifold is realized as an embedded submanifold of Euclidean space and an intrinsic one based on the ``rolling'' map. It is then shown how geometric quantities (such as curvature) are reflected by the behavior of Brownian paths and how that behavior can be used to extract information about geometric quantities. Readers should have a strong background in analysis with basic knowledge in stochastic calculus and differential geometry. Professor Stroock is a highly-respected expert in probability and analysis. The clarity and style of his exposition further enhance the quality of this volume. Readers will find an inviting introduction to the study of paths and Brownian motion on Riemannian manifolds.

The second edition of An Introduction to Differentiable Manifolds and Riemannian Geometry, Revised has sold over 6,000 copies since publication in 1986 and this revision will make it even more useful. This is the only book available that is approachable by "beginners" in this subject. It has become an essential introduction to the subject for mathematics students, engineers, physicists, and economists who need to learn how to apply these vital methods. It is also the only book that thoroughly reviews certain areas of advanced calculus that are necessary to understand the subject. Line and surface integrals Divergence and curl of vector fields

Intended for a one year course, this volume serves as a single source, introducing students to the important techniques and theorems, while also containing enough background on advanced topics to appeal to those students wishing to specialise in Riemannian geometry. Instead of variational techniques, the author uses a unique approach, emphasising distance functions and special co-ordinate systems. He also uses standard calculus with some techniques from differential equations to provide a more elementary route. Many chapters contain material typically found in specialised texts, never before published in a single source. This is one of the few works to combine both the geometric parts of Riemannian geometry and the analytic aspects of the theory, while also presenting the most up-to-date research - including sections on convergence and compactness of families of manifolds. Thus, this book will appeal to readers with a knowledge of standard manifold theory, including such topics as tensors and Stokes theorem. Various exercises are scattered throughout the text, helping motivate readers to deepen their understanding of the subject.

Over the past 15 years, there has been a growing need in the medical image computing community for principled methods to process nonlinear geometric data. Riemannian geometry has emerged as one of the most powerful mathematical and computational frameworks for analyzing such data. Riemannian Geometric Statistics in Medical Image Analysis is a complete reference on statistics on Riemannian manifolds and more general nonlinear spaces with applications in medical image analysis. It provides an introduction to the core methodology followed by a presentation of state-of-the-art methods. Beyond medical image computing, the methods described in this book may also apply to other domains such as signal processing, computer vision, geometric deep learning, and other domains where statistics on geometric features appear. As such, the presented core methodology takes its place in the field of geometric statistics, the statistical analysis of data being elements of nonlinear geometric spaces. The foundational material and the advanced techniques presented in the later parts of the book can be useful in domains outside medical imaging and present important applications of geometric statistics methodology Content includes: The foundations of Riemannian geometric methods for statistics on manifolds with emphasis on concepts rather than on proofs Applications of statistics on manifolds and shape spaces in medical image computing Diffeomorphic deformations and their applications As the methods described apply to domains such as signal processing (radar signal processing and brain computer interaction), computer vision (object and face recognition), and other domains where statistics of geometric features appear, this book is suitable for researchers and graduate students in medical imaging, engineering and computer science. A complete reference covering both the foundations and state-of-the-art methods Edited and authored by leading researchers in the field Contains theory, examples, applications, and algorithms Gives an overview of current research challenges and future applications

This book focuses on the elementary but essential problems in Riemann-Finsler Geometry, which include a repertoire of rigidity and comparison theorems, and an array of explicit examples, illustrating many phenomena which admit only Finslerian interpretations. "This book offers the most modern treatment of the topic ..." EMS Newsletter.

This book provides an introduction to the differential geometry of curves and surfaces in three-dimensional Euclidean space and to n-dimensional Riemannian geometry. Based on Kreyszig's earlier book Differential Geometry, it is presented in a simple and understandable manner with many examples illustrating the ideas, methods, and results. Among the topics covered are vector and tensor algebra, the theory of surfaces, the formulae of Weingarten and Gauss, geodesics, mappings of surfaces and their applications, and global problems. A thorough investigation of Reimannian manifolds is made, including the theory of hypersurfaces. Interesting problems are provided and complete solutions are given at the end of the book together with a list of the more important formulae. Elementary calculus is the sole prerequisite for the understanding of this detailed and complete study in mathematics.

Differential geometry arguably offers the smoothest transition from the standard university mathematics sequence of the first four semesters in calculus, linear algebra, and differential equations to the higher levels of abstraction and proof encountered at the upper division by mathematics majors. Today it is possible to describe differential geometry as "the study of structures on the tangent space," and this text develops this point of view. This book, unlike other introductory texts in differential geometry, develops the architecture necessary to introduce symplectic and contact geometry alongside its Riemannian cousin. The main goal of this book is to bring the undergraduate student who already has a solid foundation in the standard mathematics curriculum into contact with the beauty of higher mathematics. In particular, the presentation here emphasizes the consequences of a definition and the careful use of examples and constructions in order to explore those consequences.

This is the third version of a book on Differential Manifolds; in this latest expansion three chapters have been added on Riemannian and pseudo-Riemannian geometry, and the section on sprays and Stokes' theorem have been rewritten.

This text provides an introduction to basic concepts in differential topology, differential geometry and differential equations. In differential topology one studies classes of maps and the possibility of finding differentiable maps in them, and one uses differentiable structures on manifolds to determine their topological structure. In differential geometry one adds structures to the manifold (vector fields, sprays, a metric, and so forth) and studies their properties. In differential equations one studies vector fields and their integral curves, singular points, stable and unstable manifolds, and the like.

Comparison Theorems in Riemannian Geometry

Unlike many other texts on differential geometry, this textbook also offers interesting applications to geometric mechanics and general relativity. The first part is a concise and self-contained introduction to the basics of manifolds, differential

forms, metrics and curvature. The second part studies applications to mechanics and relativity including the proofs of the Hawking and Penrose singularity theorems. It can be independently used for one-semester courses in either of these subjects. The main ideas are illustrated and further developed by numerous examples and over 300 exercises. Detailed solutions are provided for many of these exercises, making An Introduction to Riemannian Geometry ideal for self-study.
The purpose of this book is to bridge the gap between differential geometry of Euclidean space of three dimensions and the more advanced work on differential geometry of generalised space. The subject is treated with the aid of the Tensor Calculus, which is associated with the names of Ricci and Levi-Civita; and the book provides an introduction both to this calculus and to Riemannian geometry. The geometry of subspaces has been considerably simplified by use of the generalized covariant differentiation introduced by Mayer in 1930, and successfully applied by other mathematicians.
This book covers the topics of differential manifolds, Riemannian metrics, connections, geodesics and curvature, with special emphasis on the intrinsic features of the subject. It treats in detail classical results on the relations between curvature and topology. The book features numerous exercises with full solutions and a series of detailed examples are picked up repeatedly to illustrate each new definition or property introduced.
“General Relativity Without Calculus” offers a compact but mathematically correct introduction to the general theory of relativity, assuming only a basic knowledge of high school mathematics and physics. Targeted at first year undergraduates (and advanced high school students) who wish to learn Einstein’s theory beyond popular science accounts, it covers the basics of special relativity, Minkowski space-time, non-Euclidean geometry, Newtonian gravity, the Schwarzschild solution, black holes and cosmology. The quick-paced style is balanced by over 75 exercises (including full solutions), allowing readers to test and consolidate their understanding.
This text focuses on developing an intimate acquaintance with the geometric meaning of curvature and thereby introduces and demonstrates all the main technical tools needed for a more advanced course on Riemannian manifolds. It covers proving the four most fundamental theorems relating curvature and topology: the Gauss-Bonnet Theorem, the Cartan-Hadamard Theorem, Bonnet’s Theorem, and a special case of the Cartan-Ambrose-Hicks Theorem.
Offering some of the topics of contemporary mathematical research, this fourth edition includes a systematic introduction to Kahler geometry and the presentation of additional techniques from geometric analysis.
This text on analysis of Riemannian manifolds is aimed at students who have had a first course in differentiable manifolds.
This volume is an English translation of Sakai's textbook on Riemannian Geometry which was originally written in Japanese and published in 1992. The author's intent behind the original book was to provide to advanced undergraduate

and graudate students an introduction to modern Riemannian geometry that could also serve as a reference. The book begins with an explanation of the fundamental notion of Riemannian geometry. Special emphasis is placed on understandability and readability, to guide students who are new to this area. The remaining chapters deal with various topics in Riemannian geometry, with the main focus on comparison methods and their applications.

This book, based on a graduate course on Riemannian geometry and analysis onmanifolds, given in Paris, covers the topics of differential manifolds, Riemannian metrics, connections, geodesics and curvature, with special emphasis on the intrinsic features on the subject. Classical results on the relations between curvature and topology are treated in detail. The bookis quite self-contained, assuming of the reader only a knowledge of differential calculus in Euclidean space. It contains numerous exercises with full solutions and a series of detailed examples which are picked up again repeatedly to illustrate each new definition or property introduced. This book addresses both the graduate student wanting to learn Riemannian geometry, and also the professional mathematician from a neighbouring field who needs information about ideas and techniques which are now pervading many parts of mathematics.

An Introduction to Riemannian GeometryWith Applications to Mechanics and RelativitySpringer

Ricci flow is a powerful analytic method for studying the geometry and topology of manifolds. This book is an introduction to Ricci flow for graduate students and mathematicians interested in working in the subject. To this end, the first chapter is a review of the relevant basics of Riemannian geometry. For the benefit of the student, the text includes a number of exercises of varying difficulty. The book also provides brief introductions to some general methods of geometric analysis and other geometric flows. Comparisons are made between the Ricci flow and the linear heat equation, mean curvature flow, and other geometric evolution equations whenever possible. Several topics of Hamilton's program are covered, such as short time existence, Harnack inequalities, Ricci solitons, Perelman's no local collapsing theorem, singularity analysis, and ancient solutions. A major direction in Ricci flow, via Hamilton's and Perelman's works, is the use of Ricci flow as an approach to solving the Poincare conjecture and Thurston's geometrization conjecture.